PMM U.S.S.R., Vol. 46, No. 6, pp. 746-752, 1983.
Printed in Great Britain.

0021-8928/83 $7.50+0.00

UDC 531.36

CERTAIN CASES OF LOSS OF CONTROL OVER THE MOTION OF SYMMETRICAL SYSTEMS*

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The bifurcations of periodic motions of nonlinear systems are studied, which can lead to a state in which the change in the sign of the control parameter ceases to affect the character of the nonsymmetric motion of the system. Results of the investigation of the concrete systems (piecewise connected and with power-type nonlinearity) are given, illustrating the mechanism of the appearance of a malfunction.

It is known that, generally speaking, nonsymmetric motions can appear in symmetric systems when the symmetric motions become unstable (e.g. /1/). The domain of the parameter values in which the symmetric and nonsymmetric steady state modes are both stable, was discovered in the trigger scheme /2,3/. Analysis of a number of other symmetric systems has discovered numerous symmetric and nonsymmetric motions, and the possibility of intersection of their domains of existence /4–6/. In this connection a study of the cases in which a stable noncontrolled motion exists along with a stable controlled motion /7/, becomes important. In practice, such a situation is "potentially dangerous" since it is not discovered until the instant at which the control is lost. It is essential that the determination of the position of the region, within which loss of control may occur, in the parameter space, must be preceded by separation of the bifurcation nodes corresponding to the onset of the nonsymmetric motions.

The present paper deals with the cases of generation of nonsymmetric motions related to the loss of stability of symmetric oscillations, as well as to the violation of conditions of their existence in the piecewise connected systems. It is shown that the corresponding bifurcation boundaries of stability and the C-boundaries of the parameter space are dangerous /8/ and hysteretic. An oscillator with the displacement limiters and a system described by the Duffing equation already investigated more than once are used as examples. The nodal points at which nonsymmetric solutions with external force periods are generated and the regions of possible loss of control are specified in the parameter space.

1. Generation of nonsymmetric motions from the symmetric when stability is lost.

Let smooth surfaces s_+ and s_- exist in the phase space $x_1, \ldots, x_{n-1}$ of the nonlinear dynamic system such, that the problem of the motions in question $x(t)$ can be reduced to the study of a sequence of point transformations of these surfaces one onto the other

$$x_1 = f^+(t_1, x_0, t_0, a_+),\ x_2 = f^-(t_2, x_1, t_1, a_-), \ldots \tag{1.1}$$

Here f^+ and f^- are periodic vector functions $2T$-periodic in t, in which $f_n^+ = 0$, $f_n^- = 0$ represent the equation of the surfaces s_+, s_- $(x_n(t) \equiv 0)$ and a_+, a_- are parameters. We pass to the new parameters a and μ

$$a_+ = a + \mu,\ a_- = -a + \mu$$

Let the following relations hold at $\mu = 0$:

$$f^-(t_{k+1} + T, -x_k, t_k + T, -a) = -f^+(t_{k+1}, x_k, t_k, a) \tag{1.2}$$

The above relations can naturally be called the conditions of symmetrizability, since in this case symmetric solutions in the sense $x(t + T) = -x(t)$ are possible in the system. To study the fixed point $x_0 = x_2 = x_*$, $t_0 = t_2 - 2T = t_*$ of the transformations (1.1) corresponding to a symmetric solution, it is sufficient to consider the first transformation of (1.1), complementing its conditions with

$$x_0 = -x_1,\ t_0 = t_1 - T \tag{1.3}$$

Thus the coordinates x_*, t_* are given by the equations

$$-x_* = f^+(t_* + T, x_*, t_*, a) \tag{1.4}$$

and the stability is determined by the distribution, with respect to a unit circle, of the roots of the characteristic polynomial

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*Prikl.Matem.Mekhan.,46,No.6,pp.931-939, 1982

$$\det F(\lambda) = 0 \tag{1.5}$$

where $F(\lambda)$ is a square matrix the elements of which are equal to the values of the corresponding partial derivatives at the fixed point

$$\varphi_{ij} = \frac{\partial f_i^+}{\partial x_{j0}} + \delta_{ij}\lambda, \quad \varphi_{in} = \frac{\partial f_i^+}{\partial t_0} + \lambda \frac{\partial f_i^+}{\partial t_1} \tag{1.6}$$
$$i = 1, \ldots, n; \quad j = 1, \ldots, n-1$$

In what follows, we shall regard the solution (1.4) as the reference supporting solution Γ_* and introduce, for every point x_k of the sequence (1.1), new variables ξ_k, τ_k

$$x_k = (-1)^k x_* + \xi_k, \; t_k = t_* + kT + (-1)^k \tau_k \tag{1.7}$$

Taking into account (1.2), we write the equations (1.1) in the new variables in the form

$$\xi_1 = x_* + f^+(t_* + T - \tau_1, x_* + \xi_0, t_* + \tau_0, a + \mu) \tag{1.8}$$
$$-\xi_2 = x_* + f^+(t_* + T + \tau_2, x_* - \xi_1, t_* - \tau_1, a - \mu)$$

The expressions (1.8) differ only in the signs of the variables ξ, τ and μ. Therefore if Γ_* is known, then the coordinates of the point sequence generated by the phase trajectories of the nonsymmetric system can be found using only the first system of equations of (1.8), with the signs of ξ, τ, μ reversed successively. Thus the periodic solution the trajectory of which is obtained by joining two segments, is determined by the system of equations (1.8) supplemented by the conditions $\xi_2 = \xi_0, \tau_2 = \tau_0$, or by the condition which can be written more simply as $\xi_2 = \xi_0$, provided that the quantity τ is formally understood to be the n-th coordinate of the vector ξ

We study the bifurcation transitions by considering the motions close to Γ_*. In addition to the parameter μ characterizing the lack of symmetry in the system, we introduce the parameter ε by setting $a = a_* + \varepsilon$, the change in which does not affect the symmetry. We note that when a symmetric solution $(\varepsilon \neq 0, \mu = 0)$ different from Γ_* is investigated, the coordinates ξ_0, ξ_1 satisfy, according to (1.3), (1.7), the relation

$$\xi_0 = -\xi_1 \tag{1.9}$$

Let the right-hand sides of equations (1.8) admit, for $|\mu|, |\varepsilon| \ll |a|$, expansion into series in the neighborhood of x_*, t_*, a_*. We rewrite these equations in linearized form and transform them, treating the sums $\xi_0 + \xi_1$ and differences $\xi_0 - \xi_1$ as the unknowns:

$$F(-1) \times (\xi_0 + \xi_1) = -2A\mu, \; F(1) \times (\xi_0 - \xi_1) = -2A\varepsilon \tag{1.10}$$
$$(A = \partial f^+ / \partial a)$$

Here $F(\pm 1)$ represents the matrix (1.6) for $\lambda = 1$ and $\lambda = -1$, respectively. If $\det F(-1) \neq 0$, then from the first system of equations of (1.10) it follows that when $\mu = 0$, i.e. in the symmetric system, there exists only a symmetric motion. In the nondegenerate case $\det F(\pm 1) \neq 0$ we can assume that $\varepsilon = 0$ without loss of generality. Then the linearized equations (1.10) determine the unique nonsymmetric solution

$$\xi_* = \xi_0 = \xi_1 = -F^{-1}(-1) A\mu \tag{1.11}$$

This implies that, firstly, a single first system of equations of (1.8) supplemented by the relation (1.11) is sufficient for determining a nonsymmetric solution in the coordinates ξ and τ, with an error of μ^2. Secondly, a change in the character of the nonsymmetric motion of the system, i.e. the change in the sign of the solution (1.7), can have two causes, namely the change in the sing of the parameter μ and the passage of the root of the characteristic equation (1.5) through the value $\lambda = -1$, since

$$\left(\frac{\partial \xi_*}{\partial \mu}\right)_{\lambda < -1} \times \left(\frac{\partial \xi_*}{\partial \mu}\right)_{\lambda > -1} < 0 \tag{1.12}$$

The degenerate case $\det F(-1) = 0$ corresponds to the existence, at $\mu = 0$, of nonsymmetric solutions $\xi^\circ = \xi_0 = \xi_1 \neq 0$ which are close to the reference solution. In this case (1.10) becomes insufficient for the determination of the coordinates of the first system of linearized equations. Let the rank of the Jacobian of the first system of (1.8) be equal to $n-1$ when $\mu = 0$, and let us assume that when $\mu = 0, a = a_* + \varepsilon$, then any, e.g. the first $n-1$ equations will be sufficient to express the unknown coordinates of the nonsymmetric solution as functions of τ° and ε. We substitute $\xi^\circ(\tau^\circ, \varepsilon)$ into the last equation

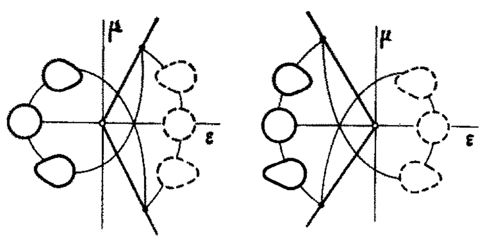
$$f_n^+ (t_* + T - \tau^\circ, x_* + \varepsilon^\circ (\tau^\circ + \varepsilon), t_* + \tau^\circ, a_* + \varepsilon) = 0 \tag{1.13}$$

and write it in the form of a series in τ° and ξ. In the general case of the degeneration in question, the first nonzero terms will be those containing $\tau^{\circ 2}$ and ε. The condition of existence of the solution $\tau^{\circ 2} > 0$ sought has the corresponding expression

$$\operatorname{sign} \varepsilon = -\operatorname{sign} \left(\frac{\partial^2 f_n^+}{\partial \tau^2} \times \frac{\partial f_n^+}{\partial a} \right) \tag{1.14}$$

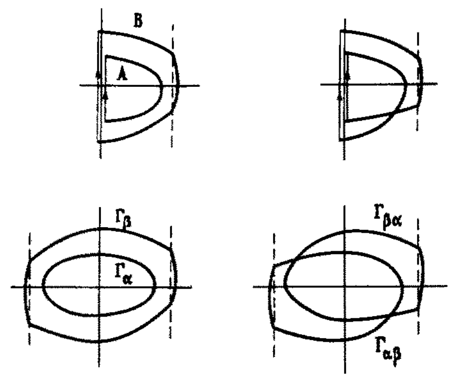
It follows that the change in ε during the passage through $\varepsilon = 0$ is accompanied, in the phase space, by either an appearance or disappearance of two nonsymmetric periodic solutions. As regards the symmetric solution (1.9), it exists according to the second system of (1.10), for both $\varepsilon > 0$ and $\varepsilon < 0$. Thus the boundary $\varepsilon = 0$ separates the region of existence of a symmetric solution from the region in which, in addition to the symmetric solution, a pair of nonsymmetric solutions also exists. The boundary between those two regions in the ε, μ parameter plane must be, generally speaking, irreversible (hysteretic) by virtue of the condition (1.12). If the degeneration $\det F(-1) = 0$ is connected with the symmetric solution's loss of stability, then either two stable nonsymmetric solutions (Fig.1a) appear at the node $\varepsilon = \mu = 0$, or the merger takes place with the pair of unstable nonsymmetric solutions (Fig.1b).

2. Generation of nonsymmetric motions at C-bifurcations. Let the phase half-space of the symmetric system be composed of regions of the same dimension Φ_α, Φ_β, and let the phase trajectory Γ_α be symmetrically distributed in Φ_α. On changing the parameter a the C-bifurcation of the motion Γ_α is accompanied by the arrival of its trajectory at the boundary of the region Φ_β and merger with the phase trajectory of the symmetric periodic motion Γ_β containing the part of the motion appearing in the region Φ_β. The fixed points corresponding to $\Gamma_\alpha, \Gamma_\beta$ are determined using the "shortened" equations of the transformations generated by the phase trajectories, on the half-period of the motions only



a b

Fig.1



a b

Fig.2

$$x_1 = f_\alpha (t_1, x_0, t_0, a), \; x_1 = f_\beta (t_1, x_0, t_0, a) \tag{2.1}$$

and artificially closed by the conditions (1.3). The stability of the fixed points is determined by the spectrum of the eigenvalues α_i, β_i.

We shall denote the "semiperiodic" motions by A, B if they are stable, and by α, β if they are unstable, and the total motions $\Gamma_\alpha, \Gamma_\beta$ we denote by AA, BB, $\alpha\alpha$, $\beta\beta$ (Fig.2). At the C-birfucation the semiperiodic motions coincide, therefore the conditions used in determining the structure of the C-boundary /6/ require the knowledge of the total number of real values of α_i and β_i situated to the right of $+1$ (σ_+) and left of -1 (σ_-).

1°. If σ_+ is even, then both semiperiodic and complete motions transform into each other. The following transition patterns are possible:

$$AA \to BB, \; AA \to \beta\beta, \; \alpha\alpha \to \beta\beta \tag{2.2}$$

2°. If σ_+ is odd, then the semiperiodic motions merge with complete motions and vanish. Two cases are possible

$$AA, \beta\beta \to \varnothing, \; \alpha\alpha, \beta\beta \to \varnothing \tag{2.3}$$

3°. If σ_- is odd, then the simplest bifurcations shown above are accompanied by the appearance (or disappearance) of a double semiperiodic motion the trajectory of which is composed of the trajectories of the semiperiodic motions (Fig.2b) and corresponds to the nonsymmetric periodic motion. By virtue of the symmetry of the initial mathematical model the order in which the semiperiodic motions are joined, is equivalent. Consequently a pair of nonsymmetric periodic motions AB, BA or $\alpha\beta$, $\beta\alpha$ (Fig.2b) appears in (or disappears from) the phase space at once. The transitions (2.2), (2.3) are accompanied by the change of sign of the expression (1.5) at $\lambda=-1$, therefore, when the control parameter μ is introduced, the dependence of the periodic solutions close to AA, $\alpha\alpha$ and BB, $\beta\beta$, on μ will have different signs and the corresponding C-boundaries of the a, μ parameter plane will be irreversible (hysteretic).

The location of the region of existence of the nonsymmetric modes relative to the C-boundary is determined by the roots λ_i of their characteristic equation, and the roots of the double semiperiodic motion formally constructed from the semiperiodic motion. Let us denote these roots by α_i^2. The motions constructed in this manner coincide in the case of C-bifurcation and conditions $1^\circ - 3^\circ$ formulated above apply to them. Thus, depending on whether the total number of real values of λ_i and α_i^2 situated to the right of $+1\,(\sigma_+')$ is even or odd the region of existence of nonsymmetric motions either coincides with the region of existence of Γ_α, or is situated on the other side of the C-boundary.

The process of determining the structure of the C-bifurcation transitions described here can in turn be applied to periodic modes obtained by joining two, four, etc. semitrajectories. Limiting ourselves to the double semiperiodic nonsymmetric motions, we find that apart from (2.2), (2.3) the following structures of the C-bifurcation transitions are possible in the symmetric systems. The change in the type of symmetric motion when a pair of nonsymmetric motions appears, takes place when σ_- is odd and σ_+, σ_+' are even. In this case the following five structures are possible:

$$\alpha\alpha \to \beta\beta, AB, BA;\ \alpha\alpha \to \beta\beta, \alpha\beta, \beta\alpha$$
$$AA \to \beta\beta, AB, BA;\ AA \to \beta\beta, \alpha\beta, \beta\alpha;\ \alpha\alpha \to BB, \alpha\beta, \beta\alpha$$

A merger followed by a disapperance of two types of symmetric motions and of a pair of nonsymmetric motions will take place when all three numbers $\sigma_+, \sigma_-, \sigma_+'$ are odd, and three transition structures are then possible

$$AA, \beta\beta, \alpha\beta, \beta\alpha \to \phi;\ \alpha\alpha, \beta\beta, AB, BA \to \phi;\ \alpha\alpha, \beta\beta, \alpha\beta, \beta\alpha \to \phi$$

3. The examination of the bifurcations of periodic motions carried out above enables us to single out several interesting features of a possible behavior of the systems in the neighborhood of the bifurcation boundaries. The first one is the high sensitivity of the motion towards the lack of symmetry in the system (parameter μ). According to (1.11), this should be expected when $|\det F(-1)| \ll 1$. This may prevent the strictly symmetric motions from being observed/9/. The second feature is the possibility of a reverse effect, namely, the detection of a "flickering" symmetric motion in the regions of the parameter values in which the symmetric motions are unstable. The flickering mode can be observed in a narrow band of existence of a pair of stable nonsymmetric motions, provided that the band is overlapped by the fluctuations in the value of the parameter μ. The third case is the possibility of loss of control over the motion of symmetric systems when the change in the sign of the control parameter μ ceases to affect the character of the nonsymmetric motion of the system.

As the first example of the system in which the loss of control over the motion may occur, we shall consider the forced vibrations of an oscillator with limiters described in the dimensionless form by the equations

$$x^{\cdot\cdot} + x = \cos\omega\tau,\ a_- < x < a_+ \tag{3.1}$$
$$x^{\cdot}(\tau+0) = -Rx^{\cdot}(\tau-0),\ x = a_+,\ x = a_-$$

where R is the velocity restoration coefficient $(0 < R < 1)$. The collisionless periodic motions represent the forced steady state oscillations of a linear system

$$p(\tau) = \frac{\cos\omega\tau}{1-\omega^2} = \varkappa\cos\omega\tau,\quad a_- < \varkappa < a_+ \tag{3.2}$$

Let us pass to new parameters $a = (a_+ - a_-)/2$, $\mu = (a_+ + a_-)/2$. We introduce the point transformations Π_R, and the transformations Π transforming the half-surface $x = 0, x^{\cdot} > 0$ onto the half-surface $x = 0$, $x^{\cdot} < 0$, generated by the trajectories of the system (3.1). The equations of Π_R include the collision at the instant $\tau = \tau_1 \in (\tau_0, \tau_2)$ and are written in the form

$$x_1^{\cdot} = -R\,(p_1^{\cdot} + p_0\sin\tau_{01} + (x_0^{\cdot} - p_0^{\cdot})\cos\tau_{01}) \tag{3.3}$$
$$p_1 - p_0\cos\tau_{01} + (x_0^{\cdot} - p_0^{\cdot})\sin\tau_{01} - a - \mu = 0$$

$$x_2^{\cdot} = p_2^{\cdot} + (p_1 - a - \mu)\sin\tau_{12} + (x_1^{\cdot} - p_1^{\cdot})\cos\tau_{12}$$
$$p_2 - (p_1 - a - \mu)\cos\tau_{12} + (x_1^{\cdot} - p_1^{\cdot})\sin\tau_{12} = 0$$
$$\tau_{ij} = \tau_j - \tau_i,\ p_i = p(\tau_i)$$

and the equations of Π are determined by a collisionless trajectory

$$x_2^{\cdot} = p_2^{\cdot} + p_0\sin\tau_{02} + (x_0^{\cdot} - p_0^{\cdot})\cos\tau_{02} \qquad (3.4)$$
$$p_2 - p_0\cos\tau_{02} + (x_0^{\cdot} - p_0^{\cdot})\sin\tau_{02} = 0$$

when $\mu = 0$, the right-hand sides of the equations satisfy the conditions of symmetrizability (1.2). The coordinates of the fixed point of the symmetric equation (stable BB_s or unstable $\beta\beta_s$), with two collisions per period $2T = 2\pi/\omega$ are obtained by complementing the equations (3.3) by conditions (1.3). In the case of C-bifurcation merger of the collision and collisionless periodic solutions we either have $x_1^{\cdot} \to 0$ for Π_R, or, in the case of Π (3.4) one of the following conditions will hold:

$$|\varkappa| = a + \mu,\ \mu \leqslant 0;\ |\varkappa| = a - \mu,\ \mu \geqslant 0 \qquad (3.5)$$

Analyzing the roots of characteristic equations at $\mu = 0$ shows that the C-boundary (3.5) corresponds to the simplest transition $\beta\beta_s \to AA$ and either separates the region of existence of the symmetric collisionless motion AA from that of the nonsymmetric subharmonic motions with a single collision per period $2T = 2^k\pi/\omega$ $(k = 1, 2, \ldots)$ within the frequency interval

$$2n + 1 < 1/\omega < 2n + 1 + 2^{-k},\ n = 0, 1, 2, \ldots \qquad (3.6)$$

or it represents a boundary at which they merge and subsequently vanish within the intervals

$$2n + 1 + 2^{-k} < 1/\omega < 2n + 1 + 2^{-k+1} \qquad (3.7)$$

The bifurcation pattern for the case when the nonsymmetric character of the system is taken into account, is discussed for $\omega = 0.8$, $R = 0.5$. The structure of the C-boundary (3.5) is determined by the condition (3.6) for $n = 0$, $k = 1$, and by the roots of the characteristic equation of the subharmonic, single collision solution of the order of $\frac{1}{2}$. When $\mu = 0$, the structure is described, within the framework of the motions under consideration, by the transition

$$BAAA,\ AAAB,\ \beta\alpha,\ \alpha\beta,\ \beta\beta_s \to AA$$

The symmetric, two-collision motion $\beta\beta_s$ retains its instability in the interval $y = a/\varkappa \in (0.5; 1,0)$. At $y = 0.5$ it becomes stable, with the root passing through the value $\lambda = -1$. We can sharpen the structure of such a bifurcation transition in accordance with the condition (1.14), by setting $y = 0.5 + \varepsilon$. The case $\lambda = -1$ has the corresponding solution $x_*^{\cdot} = -0.381$, $\mathrm{tg}\,\omega\tau_* = -0.105$ and the values of the corresponding derivatives are $x_a^{\cdot} = -0.546$, $x_\tau^{\cdot} = 0{,}307$, $x_{\tau\tau}^{\cdot} = 1.38$, $f_a = 0.68$, $f_{\tau\tau} = 0.341$. In accordiance with (1.14) a pair of nonsymmetric solutions $\beta\beta_n$ exists when $\varepsilon < 0$, therefore we have, in the neighborhood of $y = 0.5, \mu = 0$, the transition $\beta\beta_n, \beta\beta_n, BB_s \to \beta\beta_s$ (Fig. 1b) along the axis a. The bifurcation node is a source of the limits of stability corresponding to the root $\lambda = 1$ for one of the motions $\beta\beta_n$ when $\mu < 0$, and for the other motion when $\mu > 0$.

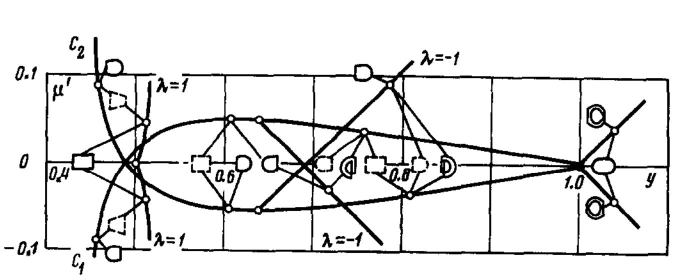


Fig.3

The analysis of the nonsymmetric motions of the type $\beta\alpha, \alpha\beta$ yields the following equations for the boundaries of stability ($\lambda = -1$): $a + \mu = 0.691$ if $-a + \mu < x(\tau)_{\min}$ $-a + \mu = 0.691$ when $a + \mu > x(\tau)_{\max}$. The boundaries shown extend to their intersections with the C-boundaries of the C_1 or C_2 apperance of additional contact between the trajectories of motion and the second limiter $x = -a + \mu$ or $x = a + \mu$. The boundaries C_1 and C_2 originate at the node $y = 1$, $\mu = 0$ and intersect each other and the axis a at the node $y = 0.495$. The structure of the boundaries in the parameter space is determined by the roots of the characteristic equations of soltuions of the type $\beta\beta_n, AB, \alpha\beta$. If a single collision motion is stable, then we have the transition $\beta\beta_n, AB \to \oslash$, and we have $\beta\beta_n, \alpha\beta \to \alpha\beta\beta\beta$ if it is unstable.

Fig.3 depicts the separation of the y, $\mu' = \mu/\varkappa$ parameter plane into regions containing various periodic motions, and the structures of the separated bifurcation boundaries are shown within the framework of the periodic solutions under consideration. In the interval $0.8 < y < 1.0$ the symmetric solutions are unstable, the boundaries C_1 and C_2 which determine the region of existence of two stable nonsymmetric solutions, are situated in a narrow strip $|\mu'| < 0.03$. If the fluctations of μ' overlap the strip in question, a flickering symmetric motion can be expected. Such motion is indeed observed when the behavior of the system is simulated on an analog computer.

The control over the motion can be lost in the interval $0.48 < y < 0.52$. Fig.4 depicts the relations

$$J(\mu') = \frac{1}{4\varkappa} \int_0^{2T} x(\tau, \mu')\, d\tau$$

for some values of the parameter y obtained analytically and by modelling on a digital computer. The case of $y = 0.505$ should be treated separately, since when $\mu' = 0$, we have several nonsymmetric stable solutions while the symmetric solution is found to be unstable. The latter is easily discovered and the problem of control can be solved by choosing the interval of variation equal to $|\mu'| > 0.05$ (similar cases occur, e.g. in investigating the control of ships). A crisis situation can still arise in the interval $|\mu'| < 0.15$ when the system passes into an uncontrollable mode of motion (indicated by an arrow).

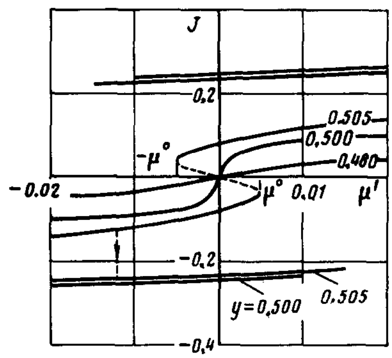


Fig.4

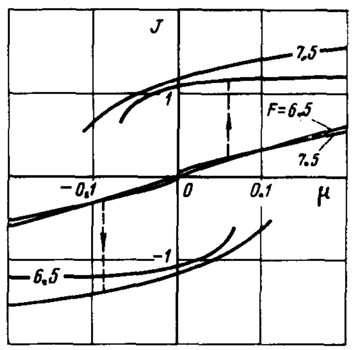


Fig.5

As a second example we consider a system with power type nonlinearity described by the Duffing equation

$$x'' + 0.2x' + x + 5x^3 = F \cos \omega t + \mu$$

The region of possible loss of control over the motion was separated by, first determining the nodes at which the nonsymmetric solutions were generated at the fundamental frequency. We shall use the qualitative pattern of the structure of the parameter space obtained in the previous example. The nonsymmetric solutions were appearing at the frequency $\omega = 0.8$ when the limiters of the oscillator displacements were moved nearer to each other. In the Duffing equation the increase in the value of F corresponds to "converging of the limiters".

Integrating the equations numerically on a computer using the Runge–Kutta method, enabled us to separate the interval of possible loss of control $5.5 < F < 8.0$ in which the nonsymmetric solutions exist. The symmetric solution is stable when $F < 6.7$. It becomes unstable with increasing $F > 6.7$, and generates an additional pair of stable nonsymmetric solutions including the solutions which are subharmonic, but sufficiently close to the symmetric solutions.

It should be noted that from the practical point of view it is often sufficient to inspect the behavior of the system at prescribed values of the parameters, without concerning ourselves with the concrete types of the motions actually taking place.

Fig.5 depicts the relation

$$J(\mu) = \int_0^{2T} x(t, \mu)\, dt$$

In the case of $F = 6.5$ loss of control is possible in the interval $|\mu| < 0.07$, and for the case $F = 7.5$ in the interval $|\mu| < 0.11$. The crisis situation is accompanied by an unexpected passage of the system into the uncontrollable mode of motion (indicated by an arrow).

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Translated by L.K.